Persistent Protests

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Abstract. A continuum of citizens with heterogeneous opportunity costs participates in a protest with well-defined demands. As long as the government does not concede, it pays a cost increasing in time and participation. Citizens who are part of the victory team enjoy a "merit reward". *Every* equilibrium with protest displays: a *build-up stage* during which citizens join the protest but the government ignores them; a *peak* at which the government concedes with positive probability; and a *decay stage* in which the government concedes with some density, and citizens continuously drop out. The set of equilibria is fully described by the peak time.

1. Introduction

Public protests and social movements vary in size and duration. Static theories capture the essential multiplicity of "protest equilibria," giving us some idea of how people overcome coordination barriers (Bueno de Mesquita, 2010; Boix and Svolik, 2013; Edmond, 2013; Morris and Shadmehr, 2018). However, such theories do not capture the dynamics of protest: the entry and exit of citizens into the movement, the resulting path of the participant stock, and the pattern of government concessions over time. The objective of this paper is to study the dynamics of participation in public protest to make sense of some empirical regularities, in a context in which agents have heterogeneous opportunity costs of participating. I study how heterogeneity influences social behavior and shapes the overall contours of a persistent protest.

I understand a protest event as the gathering of people to demonstrate against some authority about a given policy. The word *persistence* in this context refers to the duration of political unrest, in each of its potentially distinct phases. A protest may take time

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to build. It may take time to die out. The government may take more or less time to concede. Perhaps the most prominent example of a persistent protest is the Arab Spring, which began in Tunisia in the early 2010s, and spreading to other countries (Pearlman, 2013; Acemoglu, Hassan and Tahoun, 2018). More recent examples of protests include Chile and Iran in 2019, where again there was persistence of the protest in its different phases. The *Black Lives Matter* movement in the US is the most recent case of public protests characterized by persistent participation in all states, with different dynamics of participation and concessions.

In this paper, I build a model of protests to capture these dynamics, including buildups, sudden or slow concessions, and decays.[1] I purposely propose a model in a simple and abstract form. With this simplicity, I aim to construct a framework representing the main forces driving protests' dynamics. The reader might miss, at first, some natural features of demonstrations in the baseline model, such as the possibility of repression (Davenport, 2007; Siegel, 2011; Yanagizawa-Drott, 2014; Shadmehr and Boleslavsky, 2020), elections (Little, Tucker and LaGatta, 2015) or ideological conflict (Battaglini, 2017). However, most of these features can be adapted into this context, as I discuss in detail in Section 5.

The following assumed features are central to my theory. First, protests are costly to both parties. For citizens, the act of protest uses time and resources. For the government, facing down a protest is costly, both in terms of economic loss and political reputation. Second, the act of participation by an individual citizen is largely voluntary.[2] Third, even if the goal of the protest is some non-excludable public good, citizens do have a separate individual incentive to participate, driven by a psychological or socially-conferred "merit reward" of being an active member of the movement. And finally, I focus on large decentralized movements in which there is no leaders nor salient agents on the citizens' side.[3] Given these features, it is natural to think of this theory as one of large protests in democracies, in which people do not seek to overturn the authorities but instead, they gather in the streets to ask for a policy change.

Formally, I posit a continuum of small players — the citizens — and a single large player, the government. Time is continuous, and at any instant citizens face a binary choice: whether to participate in a protest or not. The cost of participating is the opportunity cost of the time spent in the protest, which is heterogeneous across citizens. At any instance

[1] Tarrow (1993), Acemoglu and Wolitzky (2014), Chen and Suen (2016), and others study protest dynamics as a series of interrelated events to understand the connection between past and future unrest. I take a different approach by focusing on a specific protest and studying its evolution over time.

[2] There could be other settings in which an institutional affiliation enforces participation, but I do not study them here.

[3] As the reader will see, this decentralization has critical effects on equilibrium analysis. Not allowing atomic citizens avoids unnatural situations, such as equilibria in which the government condition its behavior on a single citizen within a crowd.

the government decides whether to concede or not, but as long as it does not concede, it faces a cost that is increasing both in the number of people protesting and in the duration of the protest (DeNardo, 1985; Wood and Jean, 2003; Chenoweth, Stephan and Stephan, 2011). At the same time, concession is also costly to the government, because in that event it has to pay the cost of some public good: a new policy perhaps, or a regime change, or an expansion of rights (McAdam and Su, 2002; Klein and Regan, 2018). Everyone can enjoy this public good, whether or not they participated in the protest.

As already mentioned, citizens additionally enjoy a reward for being actively involved in the protest if and when the government concedes. This reward is mainly thought of as a psychological component (Blattman and Miguel, 2010; Pearlman, 2013; Passarelli and Tabellini, 2017; Aytaç and Stokes, 2019; Bueno de Mesquita and Shadmehr, 2022), but one could also interpret it as either a moral obligation (Kuran, 1990; Opp, 1994) or some material non-exclusive reward (i.e., a probability of getting a seat at the table, which depends on persistent participation and involvement). To emphasize that the duration of involvement matters, I refer to this one-time victory payoff as a *veteran prize*. This formulation is close to "value-expectancy" models (Rasler, 1996), and it aims to combine an instrumental motive, i.e. obtaining the public good, with an intrinsic motive, i.e. personally contributing to the victory. The veteran reward increases with the time spent in the protest, but is only made available once the government concedes. The model works with the same qualitative features whether or not the reward is fully contingent on being there at the moment of victory, but for concreteness I focus on this particular case.

In this dynamic game, one side is populated by a continuum of agents. As it is natural in standard policy analysis, I assume that the government can only observe citizens' aggregate behavior. Then, every aggregate strategy that is the same barring a measure zero of agents will be taken to generate the same observed history from the point of view of the government. (Matters would be different if there were a leader or a distinguished, non-anonymous agent, leading to the possibility of folk-theorem-like arguments. I do not consider that model here.)

That said, anonymity does not eliminate multiplicity, for multiplicity is a natural (and non-technical) consequence of any game with strategic complementarities. But it dramatically sharpens the set of equilibria. There is always an equilibrium with no protest and no government concession, but more remarkably, *every* equilibrium in which a protest occurs has exactly the same qualitative features. It is characterized by three stages: a build-up stage, a peak, and possibly a decay stage. The *build-up stage* corresponds to an initial period during which the protest grows as people continuously enter. It involves no concession at all on the part of the government. The second stage lasts but an instant, and is distinguished by the possibility of a government concession with positive probability —

the protest is costly enough that the government can no longer ignore it. I call this stage the *peak*. If a concession does not occur, the third and final *decay phase* starts up. It is described by continuous dropout by the citizens, with the aggregate mass of protestors shrinking with time. All along, the government concedes with a continuous but changing hazard rate that I fully characterize.

The decay stage will be familiar to any economic theorist: it unfolds as a war of attrition, but the twist I add is that one side we have a *continuum* of players; namely, the citizens (Fudenberg and Tirole, 1986; Hendricks, Weiss and Wilson, 1988; Alesina and Drazen, 1991; Hörner and Sahuguet, 2011). Their cost heterogeneity allows me to purify their aggregate behavior, leading to ongoing dropouts in the decay phase. On the other side we have the government, which must randomize according to a continuous distribution over concession times. In particular, it must be indifferent at any time between conceding and waiting another instant. For this indifference condition to hold in equilibrium, the government will concede at some time-varying hazard rate that generates exactly the path of participation rates that guarantees this indifference. As far as citizens are concerned, they take as given the hazard path, and drop out as their expected gains from continuation become too low relative to their cost. Individual exits are deterministic, and aggregate to a smooth path of decay.

The peak stage is special because it involves a non-trivial probability of concession. It is conceptually important because it suggests a sudden change in government attitudes that occurs precisely at the height of the protest. Mathematically, it "initializes" the starting conditions of the war of attrition to follow.

In addition to these features, the build-up phase I describe is, to my knowledge, completely novel. It is not a part of any war of attrition, and stems from the assumption of varying opportunity costs of participation, along with the structure of the veteran reward. Individuals enter the protest in a spread-out way, leading to a swelling in unrest. During this entire period, I show that there cannot be a positive response from the government, because it must strictly prefer not to concede in this phase.

Taken together, these phases generate a rich but uniform prediction for the path of protests. Moreover, the three stages characterize an equilibrium that unfolds as a war of attrition with endogenous payoffs that are fully characterized by the time of the peak.

A central feature of equilibrium is that individual entry and exit decisions are monotone in their opportunity costs. I show that citizens enter at most once and exit at most once. The time at which an individual enters the protest increases with her opportunity cost, and the time at which she exits decreases in her cost. The resulting dynamics of entry and exit are therefore of the first-in-last-out form. The agent with the lowest opportunity cost is the first to enter, and will hold against the government until the government has conceded. The last agent who joins the protest enters right before the peak, and exits just after it.

While build-up times, peak concession probabilities, decay rates and concession rates vary across equilibria, *all* equilibria share these qualitative features. Moreover, indiscriminate variation is not possible. I show that the set of all equilibria is fully described by a single "pseudo-parameter," the protest peak time, which can only vary within a range that I fully characterize.[4] This range is a bounded interval with a strictly positive lower bound. The veteran reward is responsible for the positive lower bound, as agents need time to build it, which means that every equilibrium with protests will involve a minimum delay before concessions are made. On the other hand, the peak is also bounded above, so that citizens with the lowest opportunity cost have incentives to begin the protest. Overall, the equilibrium characterization develops as a war of attrition with endogenous payoffs: the hazard rate at which the government concede and the payoffs to the agents of dropping out will depend on the equilibrium peak time.

These predictions highlight the relevance of analyzing the dynamic shape of protests, not just theoretically but empirically. Specifically, my model provides three main empirical predictions. First, participation is *single-peaked*, as there is always an initial period of build-up in participation and then, possibly, some decay. Second, citizens' strategies are monotone in their opportunity costs. And, third, concessions by the government should occur at the peak of the involvement or *after* it. None of these predictions is trivial. Single-peakedness precludes the existence of waves or backlashes in participation. The effect of opportunity costs over the timing of participation—although intuitive—is critical in a framework with heterogeneous agents since it allows us to understand the identity of the agents protesting at any time. It also allows characterizing the relationship between opportunity costs and the duration of protests (Chassang and Padró i Miquel, 2009; Dal Bó and Dal Bó, 2011; Mitra and Ray, 2014). And finally, concessions occurring in the decreasing part of participation's trajectory imply that the government optimally concedes when people are already dropping out, which might not be optimal in other dynamic situations. I discuss in more detail how this prediction differentiates the main force driving single-peakedness in this model from other possible sources.

This paper is organized as follows. In the next subsection we briefly review the related literature and my main contribution. In Section 3, I develop the baseline model and provide some discussion of its main features. In Section 4, I characterize the dynamics of protests in equilibrium. I develop some extensions in Section 5, and I show some

[4]To be precise, there is also a second "pseudo-parameter" that could index equilibria, which is the start time of the protest, but without any loss of generality I normalize this to zero.

additional properties of the equilibrium set in Appendix A. All proofs can be found in Appendix B.

2. Related Literature

This paper contributes to the literature on the dynamics of participation in public protests as a collective action problem. The literature most closely related to my paper is that studying the coordination problem among citizens. Static models of coordination in protests have been studied by Shadmehr and Bernhardt (2011), Boix and Svolik (2013), Edmond (2013), Morris and Shadmehr (2018) and Bueno de Mesquita and Shadmehr (2022).

This paper's main contribution to the literature is to characterize the dynamics of participation and government concessions in equilibrium. To the best of my knowledge, this is the first work providing a full characterization of the dynamics of protests including both the citizens and the government as strategic players. The dynamics I obtain are precisely a result of this feature of the model: it is the mutually reinforcing strategic interaction *between* the citizens and the government what generates the singlepeakedness in participation. The build-up phase is generated from the side of citizens who enter in a continuous increasing order. The decay phase comes from the side of the government: as the protest is getting costly, the government has to make concessions to encourage people to drop out.

In a related paper, Chenoweth and Belgioioso (2019) show empirical evidence of dynamics that are similar to the build-up stage in my framework. They model this by applying the momentum equation: social movements compensate for low popular support by concentrating their activities over time. As I show in Section 4.2, there is a similar trade-off in this dynamic model. In particular, within the set of equilibria, there is an inverse relationship between the rise in participation and the time the government makes the first probabilistic concession. In more recent work, Enikolopov et al. (2020) propose a model of participation dynamics in public protests, in which people's participation is motivated by social image concerns. In a context in which the government behavior is taken as given, they obtain that participation is decreasing.

The dynamics I focus on differ from those analyzed by Acemoglu and Wolitzky (2014) as I do not focus on how a current protest affects the probability of occurrence of future events. Instead, I focus on one protest and study the participation dynamics for that specific movement. Each equilibrium represents a unique protest with different stages of participation, concessional peaks, and decay.

Another strand of the literature studies participation in collective action and the effects of social interactions (Barbera and Jackson, 2019; González, 2020; Bursztyn et al., 2020). My paper differs in that I focus on the coordination game's equilibrium among citizens and the government. However, even though citizens' payoffs in my paper are not directly a function of other protesters' decisions, the latter has an indirect effect through the probability of government concession generating strategic complementarities: higher participation forces an earlier concession, and hence increases incentives for people to participate.

From a methodological point of view, this work contributes to the literature on wars of attrition. Two features make this war of attrition special. First, the decay stage of equilibria unfolds as a war of attrition with complete information between a single large player and a continuum of citizens. It is like a two-player war of attrition with complete information—as in Hendricks, Weiss and Wilson (1988)—in which one of the sides is replaced by a continuum of anonymous citizens that, when aggregated, resemble the behavior of a single opponent. And second, this is a war of attrition with endogenous payoffs: they depend on the delay in the responsiveness of concessions by the government. This type of model could also apply to other situations, such as wildcat strikes and political campaigns. Wars of attrition have been applied to understand delays in fiscal adjustments programs (Alesina and Drazen, 1991), the relationship between monetary and fiscal authorities (Backus and Driffill, 1985; Tabellini, 1988), firms' exit (Fudenberg and Tirole, 1986), and labor strikes (Kennan and Wilson, 1989).

There are other works studying wars of attrition with more than two players. Bulow and Klemperer (1999) analyze a war of attrition with a finite number of firms competing for a set of prizes. Kambe (2019) studies a war of attrition with several agents, in which the exit of a single player is enough to end the game. The lack of anonymity in these cases changes the strategic problem in ways that are unrelated to the setup analyzed here.

This work is also related to the literature studying people's motivations to protest. In particular, the literature on the social psychology of public protests studies intrinsic motives for participation as a result of ideology or group identity (Cohen (1985) and Jasper (1998)). The *veteran prize* constitutes a new explanation for persistent participation in a protest, which combines both an *intrinsic* motivation—i.e., the veteran reward—with an *instrumental* motivation—agents obtain this value only if the movement is successful.[5] In that sense, it is closer to value-expectancy models, as the one studied by Rasler (1996). Other works allowing for psychological motives to unrest include Passarelli and Tabellini (2017), Wood and Jean (2003), Pearlman (2018), and, more recently, Bueno de Mesquita and Shadmehr (2022).

Recent studies focus on agents' motives to participate in revolts. Cantoni et al. (2019) conduct a field experiment in the context of Hong Kong's anti-authoritarian movement.

[5]See Feather and Newton (1982) and Klandermans (1984) for an analysis of instrumental motivations in protests.

They obtain evidence of strategic substitutability between agents' decisions. Bursztyn et al. (2020) study the causes that sustained participation in social movements. They find that changes in the population affect people's beliefs about success, giving rise to strategic substitutability. In contrast, changes in friends' participation affect the social utility derived from the protest and give rise to strategic complementarity.

This work is also related to the literature on conflict (see Ray and Esteban (2017) for a detailed review). There is extensive literature analyzing the relationship between conflict intensity and income, the main idea being that income affects both the size of the prize that can be obtained from conflict and the opportunity costs (Chassang and Padró i Miquel, 2009; Dal Bó and Dal Bó, 2011; Mitra and Ray, 2014).

3. A Model of Protest Dynamics

In Section 3.1, I describe the baseline model, along with its main assumptions and the equilibrium concept. In Section 3.2, I comment on the assumptions and more generally on the model setup.

3.1. The Model. There is a single large player, the *government*, and a continuum of small players, the *citizens* or the *people*. Citizens are indexed by $i \in [0,1]$. Time is continuous, and at any instant $t \in [0,\infty]$, citizens decide whether to participate in a protest to ask the government for a public good. The choice for the government is also binary. At any moment in time, the government can either concede or keep waiting. The game ends when one of the two sides fully concedes: either the government provides the public good, or all citizens drop out.

Protests are costly to everyone. For citizens, participating in the protest requires an investment of time and resources, which is captured by an opportunity cost parameter θ. I assume that the opportunity cost is heterogeneous and drawn from a distribution F. In practice, this heterogeneity in opportunity costs may capture different levels of income, types of jobs, or even different residence locations that make protesting more costly for some agents than for others. I assume that F is continuously differentiable, with full support $[\underline{\theta},\overline{\theta}]$, for some $\underline{\theta} > 0$. The maximum cost $\overline{\theta}$ might be unbounded.

For the government, staring down a protest is also costly. This cost might represent losses due to direct disruption caused by demonstrations, a loss in nationwide economic productivity, or a hit to the government's political reputation (DeNardo, 1985; Hadzi-Vaskov, Pienknagura and Ricci, 2021; Gillion, 2013). I model this by assuming that the government pays a flow cost that is increasing both in the number of people participating in the protest at a given time and in the duration of the protest. Concession is also costly, as once the government concedes, it pays the equivalent of a flow cost of q forever.

Let π_t be the mass of citizens protesting at t, and let $t = 0$ be the time at which the protest begins. I make some natural assumptions regarding the cost function. First, if there is no one protesting, there is no cost to the government. Second, if the entire population is protesting, the flow cost of bearing the protest is higher than the flow cost of the public good. I summarize this and the above discussion in the following assumption.

Assumption 1. *The cost function $c : [0,1] \times [0,\infty) \to \mathbb{R}_+$ is continuously differentiable on both arguments and satisfies:*

(i) $c(0,t) = 0$ for all t, and $c(1,0) > q$;

(ii) $c(\pi,t)$ is strictly increasing in π, and is strictly increasing in t if $\pi > 0$.

Let $(\pi_t)_{t\geq 0}$ be a trajectory of participation. If the government concedes at some time τ, then its overall costs are given by:

$$\int_0^\tau e^{-rs} c(\pi_s, s) ds + e^{-r\tau} \frac{q}{r}, \tag{1}$$

where $r > 0$ is the discount rate, which is the same as the citizens' discount rate.

Because the public good is non-excludable, even citizens who did not protest can enjoy it. If the government concedes at a time τ, then from that time onward, every citizen receives an extra flow payoff from enjoying the public good. Notice that the value of the public good does not affect citizens' decision to protest, and it is without loss to assume that all of them obtain a value from the public good equal to 1.

In addition to the payoff from the public good, citizens get a reward for being active participants in the protest. This payoff increases with the time spent in the protest, and it is made available only if the citizen is still protesting by the time the government concedes. I call this prize the *veteran reward*. Formally, if the government concedes at time t, an agent who has been in the protest since time t_0, and is still in the protest when the government concedes, gets a one-time reward of $v(t - t_0)$. I assume that the veteran reward increases with the time spent in the protest, but at a decreasing rate. The following assumption formalizes this idea.

Assumption 2. *The veteran reward $v : [0,\infty) \to \mathbb{R}_+$ is continuously differentiable, and*

(i) $0 < v'(\Delta) < \infty$ and $v''(\Delta) \leq 0$ for all $\Delta \geq 0$;

(ii) $v(0) = 0$;

(iii) $\int_0^1 \frac{1}{v(s)} ds < \infty$.

Part (i) ensures that v is increasing and concave. Part (ii) rules out opportunistic behavior, as it precludes the possibility of agents entering the protest at the exact moment the government is conceding. Part (iii) is more technical in nature, and it ensures the veteran reward increases "quickly" at 0.[6] As an example, a veteran reward of the form $v(\Delta) = \sqrt{\Delta}$ satisfies Assumption 2.

The idea of the veteran reward is closely related to "value-expectancy models", according to which people rebel if they are convinced that they will achieve the collective good (Klandermans, 1984; Muller and Opp, 1986). Rasler (1996) explores the ideas of value-expectancy model in the context of the Iranian revolution, and shows that government concessions increase both protest actions and the spatial diffusion of protests. Besides value-expectancy models, psychological motivations to protest have been discussed extensively in the literature on why people protest (Blattman and Miguel, 2010; Pearlman, 2013; Passarelli and Tabellini, 2017; Aytaç and Stokes, 2019).

In addition to the assumptions above, I assume the following condition, which ensures that there are some dominance regions. The idea of this condition is that even if there is a protest, there is always a mass of people large enough for whom participating in the protest is too costly.[7]

Assumption 3. *The cost function satisfies* $c(F(v'(0)), 0) < q$.

Suppose that the government concedes at some time τ, possibly random. Consider a citizen with opportunity cost θ who starts protesting at some time t_0 and is planning to exit at time t_1. Her expected payoff is given by the following expression:

$$E\left[-\theta \int_{t_0}^{t_1 \wedge \tau} e^{-rs} ds + e^{-r\tau}\left(\mathbb{1}_{\tau < t_1} v(\tau - t_0) + \frac{1}{r}\right)\right], \tag{2}$$

where the expectation is taken over τ. In words, the citizen will pay the cost of the protest for as long as she remains an active participant. If, by the time the citizen drops out, the government has not conceded, then the citizen simply goes home and receives nothing at that time. Eventually, she will get to enjoy the public good if and when the government

[6] Even though I use Assumption 2.(iii) to show the existence result (Section 4.2), note that we would still obtain existence of equilibrium without it, but this equilibrium would be degenerate.

[7] To understand the expression in Assumption 3, note that $v'(0)$ corresponds to the marginal value of entering the protest for the agent who enters last—i.e., the agent with the highest opportunity cost among all the agents that ever join the protest. Then, even if all those willing to enter the protest entered at 0, the protest would be less costly than the public good.

decides to provide it. If, on the contrary, the government concedes before the citizen drops out, then, in addition to the public good, she gets a one-time veteran reward of $v(\tau - t_0)$.[8]

It remains to specify how the game is played at each instant. I assume that when the government decides whether to concede, it is already observing how many people are protesting. However, when citizens decide whether or not to protest, they observe participation only until *an instant before* they join. To help better explain the interpretation for continuous time, we can build some intuition with a discrete-time case. Imagine a game played repeatedly at times $\{0, 1, 2, ...\}$. At any time t, the stage game is such that, first, citizens make a protest decision, and then the government decides whether or not to concede. Thus, when citizens choose their actions, they observe only a history of participation up to $t-1$—i.e. $\{\pi_0, \pi_1, ...\pi_{t-1}\}$. Once they take an action, the government gets to observe π_t before deciding whether to concede. Hence, the relevant history for the government is given by $\{\pi_0, \pi_1, ..., \pi_t\}$.

Following this intuition, for any time t, define the histories $\pi^t = \{\pi_s : 0 \leq s < t\}$ and $\overline{\pi}^t = \{\pi_s : 0 \leq s \leq t\}$. Let $\Pi^t = \{\pi^t\}_{t \geq 0}$ be the set of all possible open histories at time t, and $\overline{\Pi}^t = \{\overline{\pi}^t\}_{t \geq 0}$ the set of all possible closed histories at time t. Also, define $\pi^0 = \emptyset$. A strategy for the government is a process $\gamma = \{\gamma_t\}_{t \geq 0}$, with $\gamma_t : \overline{\Pi}^t \to \{0, 1\}$, where $\gamma_t = 1$ stands for *concede* and $\gamma_t = 0$ for *not concede*. A strategy for a citizen with opportunity cost θ is a process $\sigma^\theta = \{\sigma_t^\theta\}_{t \geq 0}$ with $\sigma_t^\theta : \Pi^t \to \{0, 1\}$, where 1 stands for *participate* and 0 for *not participate*. While the government decision is irreversible, citizens can reenter the protest after leaving. We denote a strategy profile by (σ, γ), where $\sigma = \{\sigma^\theta\}_{\theta \in [\underline{\theta}, \bar{\theta}]}$.

For any strategy profile (σ, γ), let $\pi^{\sigma t}$ be the trajectory up to time t, conditional on no concession, generated by the strategy σ.[9]

I focus on the set of Nash Equilibria of the game. Given that the only observable that matters in equilibrium is the aggregate behavior of protesters and not their individual decisions, citizens are anonymous (**?**). Then, it is enough to describe the government's strategies along the equilibrium path.[10]

I allow the government to randomize over concession times. As we focus on the trajectory of participation that the government expects in equilibrium, we can characterize its

[8] As I discuss below, the model could be modified to allow citizens to receive the veteran reward even if they drop out before the government concedes.

[9] This can be defined recursively as follows:

$$\pi_t^\sigma = \int \sigma_t^\theta(\pi^{\sigma t}) dF(\theta) \qquad \forall t \geq 0. \tag{3}$$

[10] This is equivalent to focusing on government strategies that are *open-loop*, in the sense that it is as if the government commits to a sequence of actions at the beginning of the game. Fudenberg and Levine (1988) compare the notions of *open-loop* and *closed-loop* equilibria for the case of games with non-atomic players.

strategy as a mixed strategy: a distribution of concessions $G(t)$.[11] This distribution of government concessions corresponds to the probability of the government conceding in $[0, t]$, given a trajectory of participation up to time t, $\pi^{\sigma t}$. This function is weakly increasing and right continuous in t, with support:

$$\mathcal{T} = \{t \geq 0 | G(t) - G(t-\epsilon) > 0 \ \ \forall \epsilon > 0\}. \tag{4}$$

Define $\tau_0 = \inf \mathcal{T}$—i.e., the first time at which the government makes some concession—and $\tau_1 = \sup \mathcal{T}$. On the citizens' side, their anonymous nature comes into play again, as it implies that we can obviate mixed strategies and focus on pure strategies only.

I focus on the set of Nash Equilibria of the game. An equilibrium is given by a distribution of government concessions $G(t)$ and a profile of citizens' strategies σ, such that given the outcome path $\{\pi_t^\sigma\}_{t\geq 0}$,

(i) the government's strategy maximizes its expected total payoff; and

(ii) citizens' strategies maximize their expected total utility given the government's distribution of concession G.

3.2. Key Features of the Model.

The Psychology of the Veteran Prize.— Scholars have explained participation in these collective action problems either by introducing incomplete information (Lohmann, 1993; Diermeier and Van Mieghem, 2008; Gerardi et al., 2016; Battaglini, 2017; Barbera and Jackson, 2019) or by introducing some intrinsic payoffs (Wood and Jean, 2003; Pearlman, 2018). The veteran prize follows the second line, and aims to capture a complementarity between instrumental and intrinsic motives.[12] People want to have merit in an eventual victory against the government. The necessity of the victory captures the instrumental component, whereas the merit captures the intrinsic component. As the protest needs persistence to be successful, merit is increasing in participation time, and so it is the veteran reward.

Conditional Nature of the Veteran Prize.— As the model is currently described, citizens get their veteran prize only if they are actively participating at the time the government concedes. It might be natural, however, for citizens who make a relevant contribution to

[11] Without anonymity, a behavioral strategy in this context would specify for each possible history $\overline{\pi}^t$, a probability of concession.

[12] The literature on the social psychology has identified four motives for protesting: (i) *Instrumental*: related to the expectation of reaching a goal; (ii) *Identity*: related to the identification with a group; (iii) *Emotions*: related to grievances and group-based anger; and (iv) *Ideology*: related to individual values and the perception of an illegitimate state of affairs. The latter three motives, generate an inner obligation to contribute that prevents free riding. However, as Simon et al. (1998) show, in practice, these three motives complement the instrumental one. See also Klandermans (1984) andVan Stekelenburg and Klandermans (2013).

building up the protest to obtain some reward, even if they drop out before the government concedes. It is direct to extend the model to allow for citizens to obtain part of the veteran prize even if they retire before concession, provided that they obtain it only when the government concedes.[13]

Cumulative Nature of the Veteran Prize.— I formulated the model in such a way that the veteran prize is a function of how long the citizen has been in the protest before government concession. This assumption can be easily modified without changing agents' behavior. As citizens discount the future and the opportunity cost is constant, they will always prefer to push all their participation forward.

Heterogeneity in Opportunity Costs.— In my model, agents' heterogeneity comes from differences in their opportunity costs. Naturally, there might be other sources of heterogeneity that are relevant in the context of public protests, such as preferences and stakes. Differences in the value from the public good do not affect citizens' decisions, and then a common value is without loss of generality. However, if these heterogeneous values affect the veteran prize's magnitude, the heterogeneity becomes relevant. In that case, the opportunity cost parameter would capture the net effect of costs and valuations.

Government's Problem.— I assume the government's decision is whether to concede or ignore the protest. However, it is not hard to think in situations where the government's choice is not binary. Frequently governments try to dissuade protesters either by making small concessions or by implementing repressive tactics. Partial concessions can be easily allowed in this model, under some modifications to the way protesters obtain their rewards. I explore this in Section 5.1 in the Appendix. The case of repression is more delicate. In the model, it can be represented by an increase in the cost of protesting. In Appendix A.3 I show that the impact of an increase in opportunity costs over the expected duration of a protest is ambiguous, and so it is the optimality of repression.

Citizens' Anonimity.—Citizens in this model are anonymous, in the sense that their behavior is only relevant to the government when aggregated. I claim this to be the most—if not the only—natural assumption in large decentralized protests. Citizens make their own participation decisions, and they lack the personal power to carry the government toward a concession state. They are influential when they are together, and hence it is not in the government's interest to focus on a single one of them. Anonimity, then, naturally rules out any equilibria in which the government conditions its behavior on a single citizen.

And by no means am I neglecting the possibility of some agents becoming distinguishable. A leader, for instance, has characteristics that make her directly relevant to governments'

[13] If this weren't the case, citizens' strategies would be completely independent of government behavior.

decision-making. Leaders' actions can be very consequential, and they have more bargaining power than a single ordinary citizen. But that is a different problem than the one I study here. Leaders are atomic players who are not present in this game. This paper focuses on understanding how large crowds show up on the streets in a persistent way, without previous coordination nor incentives from a leadership.

4. The Dynamics of Protests

4.1. Equilibrium Characterization. In this section, I fully characterize the set of equilibria in which a protest occurs. I refer to an equilibrium as an *equilibrium with protests* if there is some (possibly probabilistic) concession by the government—i.e., $\mathcal{T} \neq \emptyset$. In addition to the set of equilibria with protests characterized below, there is always an equilibrium in pure strategies in which the government never concedes and nobody protests—i.e., $G(t) = 0$ and $\pi_t^\sigma = 0$ for every t. This equilibrium arises naturally in coordination games with complete information, and in protest games, it represents many situations in which protests simply do not occur.

Naturally, there is a multiplicity of equilibria in this game. But what makes the results remarkable is that every equilibrium with protest has the same qualitative features. As Theorem 1 shows, any equilibrium with protests is characterized by three stages: a build-up stage, a peak, and, possibly, a decay stage. The *build-up* stage corresponds to the initial period in which the protest grows as people continuously enter. However, in this initial stage, the protest is still not costly enough to the government, and, thus, the government does not concede. The *peak* is the first time at which there is a possibility of concession by the government with positive probability. It coincides with the time at which participation reaches its maximum level, and the protest becomes costly enough that the government can no longer ignore it. If concession occurs at the peak, the protest ends. If it does not occur, then the *decay* stage starts. In the decay stage, citizens continuously drop out, and participation decreases. The government continues conceding with a decreasing hazard rate.

This result is formalized in the following theorem.

Theorem 1. *Let $G : [0, \infty] \to [0, 1]$, $(\pi_t^\sigma)_{t \geq 0}$ be an equilibrium with protests. Then, the following features obtain:*

(i) *There is always delay in government concession—i.e., $\tau_0 > 0$.*

(ii) *π_t^σ is continuous, increasing for $t \leq \tau_0$, and if $G(\tau_0) < 1$, decreasing for all $t \geq \tau_0$.*

(iii) *The distribution of concessions has, at most, one discrete jump at τ_0.*

(iv) *If* $G(\tau_0) < 1$, *then* $G(t)$ *is strictly increasing, continuous, and* $\tau_1 = \infty$.

(v) *The government concedes with probability* 1—*i.e.,* $\lim_{t \to \infty} G(t) = 1$.

Although I prove the result in Appendix B, I provide the main intuition here.

First, note that in any equilibrium with protests, the government's strategy is restricted to either a singleton support $\{\tau_0\}$, or an interval $[\tau_0, \tau_1]$ (see Lemma 2 in Appendix B). To see this, note that if the government stops conceding during some time interval and resumes concession later, citizens who are already in the protest will wait until the government starts conceding again. As the cost of the protest increases with time, this strategy cannot be optimal.

For the government to play a mixed strategy, it must be that along the support, the following indifference condition holds:[14]

$$c(\pi_t, t) = q \text{ for all } t \in [\tau_0, \tau_1]. \tag{5}$$

This indifference condition imposes a constraint on the number of protesters that the government is willing to tolerate. Define the *indifference participation level* $\tilde{\pi}_t$ as the trajectory of participation that satisfies equation 5 for any time $t \in [0, \tau_1]$. By Assumption 1, this indifference participation path is continuous and strictly decreasing in t. In equilibrium, the trajectory of participation on the support of $G(t)$ must coincide with the function $\tilde{\pi}_t$, and then it is decreasing.

From the indifference condition, I also conclude that it must be that the interval goes all the way to infinity—i.e., $\tau_1 = \infty$. This result follows from the government's incentives to randomize: it must be that at any time, the government is indifferent between conceding and waiting another instant. If the interval is finite, then there is a time at which the government is no longer indifferent, and the equilibrium will unravel.

Citizens, on the other hand, take the distribution of concession $G(t)$ as given and decide when to protest. Even when they are allowed to exit and re-enter many times, I show that in equilibrium, they enter and exit, at most, once. Moreover, they enter only before the government starts conceding, and they exit only afterwards.

Consider the problem of a citizen with opportunity cost θ, who enters at t_0 and exits at t_1. Since the government makes the first probabilistic concession at time τ_0, the entry and exit times must be such that $t_0 < \tau_0 \le t_1$. Let $\lambda_t = \frac{g(t)}{1-G(t)}$ to be the government's hazard rate of concession—i.e., the instantaneous probability of conceding at time t, given that it has not conceded yet. Once in the protest, this citizen keeps protesting as long as the benefit of

[14] See Lemma 3 in Appendix B.

staying another instant weakly exceeds the cost. In particular, she exits the protest if:

$$\theta \geq \lambda_{t_1} v(t_1 - t_0). \tag{6}$$

The left-hand side corresponds to the opportunity cost of staying another instant. The right-hand side corresponds to the expected gains: the veteran reward she can obtain, times the hazard rate at which the government is conceding.

Consider, now, the entry decision of the citizen who expects to exit at t_1. At any time $t < \tau_0$, she compares the expected payoff from entering at t against the payoff from waiting an instant to enter. By entering at t instead of an instant later, the agent has to pay the flow opportunity cost θ. However, the gains are given by the marginal increase in the veteran prize that the agent might obtain during the time she remains in the protest. Then, an agent with opportunity cost θ enters the protest at t_0 if:

$$\theta \leq E\left[e^{-r(\tau - t_0)} \mathbb{1}_{\tau < t_1} v'(\tau - t_0)\right]. \tag{7}$$

Citizens' utilities satisfy a single-crossing property with respect to opportunity cost, and then these optimality conditions are both necessary and sufficient.[15] Moreover, their strategies are monotone in the opportunity cost. This allows us to characterize their strategies by a pair of entry and exit thresholds that we denote by $\tilde{\theta}_0(t)$ and $\tilde{\theta}_1(t)$, respectively. At any time $t < \tau_0$, a citizen enters if $\theta \leq \tilde{\theta}_0(t)$. At any time $t \geq \tau_0$, she exits if $\theta \geq \tilde{\theta}_1(t)$. Then, equilibrium participation is given by: $\pi_t^\sigma = F(\tilde{\theta}_0(t))$ if $t \leq \tau_0$, and $\pi_t^\sigma = F(\tilde{\theta}_1(t))$ if $t > \tau_0$. The expected benefits from entry and exit depend on the government's strategy $G(t)$, and this, in turn, determines the entry and exit thresholds, $\tilde{\theta}_0(t)$ and $\tilde{\theta}_1(t)$. The entry threshold is increasing in time, which makes citizens to continuously join over time, and the exit threshold is decreasing, which makes citizens leave. In equilibrium, both thresholds coincide at τ_0, generating a continuous trajectory of participation that reaches its peak at that time.

If the government concedes with probability one on its first concession—i.e., $\mathcal{T} = \{\tau_0\}$—then there is no relevant exit decision. In this case, there is no decay stage, as the protest ends at the peak. If, on the contrary, the support $\mathcal{T}$ is an interval, then the trajectory of participation in the decay stage must coincide with the indifference participation level $\tilde{\pi}_t$. Then, the following equilibrium condition must hold:

$$\pi_t^\sigma = F\left(\tilde{\theta}_1(t)\right) = \tilde{\pi}_t. \tag{8}$$

That is, the participation level generated by citizens' best responses must coincide with the indifference participation level.

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[15]See Lemma 4 in Appendix B

The equilibrium condition allows us to pin down a precise trajectory for the hazard rate of government concession. At any time $t \geq \tau_0$, there is a citizen who is on the margin between staying another instant or dropping out. From the condition in equation 8, this citizen's opportunity cost must be such that $\tilde{\theta}_1(t) = F^{-1}(\tilde{\pi}_t)$. Then, citizens' exit times are determined in equilibrium by the trajectory of $\tilde{\pi}_t$. Given this exit time, citizens choose an entry time $t_0(t)$ according to the entry condition 7. Then, the government hazard rate at time t is given by $\lambda_t = \frac{\tilde{\theta}_1(t)}{v(t-t_0(t))}$, which defines a unique distribution of concessions $G(t)$.

4.2. Equilibrium Multiplicity. So far, I have shown that any equilibrium with protests can be parametrized by a time τ_0 at which the level of participation reaches its peak and at which the government makes the first concession. In this section, I show that the set of possible times τ_0 is bounded.

The bounds happen to be very intuitive. The lower bound, is given by the equilibrium in which the government concedes with probability 1 at the time that participation reaches its peak. Let's call this lower bound $\underline{\tau}$. If the government concedes with probability 1 at $\underline{\tau}$, the marginal benefit of the last agent entering is given by $v'(0)$, while the marginal cost is its opportunity cost, θ. As all the agents with lower opportunity cost have already entered, participation at the time of concession is given by $F(v'(0))$. Then, $\underline{\tau}$ solves $c\left(F(v'(0)), \underline{\tau}\right) = q$.

The upper bound is a bit more subtle. Recall that I normalize the time so that $t = 0$ is the time at which the first citizen enters the protest.[16] Given that entry is monotone in θ, the first citizen entering is the citizen with the lowest opportunity cost $\underline{\theta}$. Note that as the delay in the start of government concession increases, the payoff from entering at 0 also decreases. But in order to have an equilibrium with protests, at least the agent with the lowest opportunity cost must be willing to enter. Then, the upper bound $\overline{\tau}$ must be such that $\underline{\theta} = E\left[e^{-r\tau}v'(\tau)\right]$. This is, the lowest opportunity cost must equal the expected marginal benefit of entering at 0 and staying in the protest forever. This can be rewritten as:

$$\underline{\theta} = \int_{\overline{\tau}}^{\infty} e^{-rs}v'(s)dG(s), \tag{9}$$

where I have modified the right-hand side to show the direct dependence on $\overline{\tau}$. In an equilibrium in which the first discrete probabilistic concession occurs at time $\overline{\tau}$, the benefit obtained by a citizen who stays in the game forever must coincide with the lowest possible opportunity cost.

[16]To be more precise, this normalization is an equilibrium selection. However, given that protests can happen at any time, and the objective of this work is to characterize their dynamics, if we did not set the starting time to 0, the predictions obtained with this normalization could be reproduced on any possible starting point.

I impose the following assumption, which ensures that the lower and upper bounds are distinct and well defined.

Assumption 4. *Let $\underline{\tau}$ be such that $c(F(v'(0)),\underline{\tau}) = q$. Then, $\underline{\theta} < e^{-r\underline{\tau}}v'(\underline{\tau})$.*

Using these bounds, I obtain the following existence result.

Theorem 2. *For every $\tau_0 \in [\underline{\tau}, \overline{\tau}]$, there exists a unique equilibrium $(G, (\pi_t^\sigma)_{t\geq 0})$ in which the government makes the first probabilistic concession at τ_0—i.e., τ_0 is the lower bound on the support of G.*

This result provides a strong characterization of the set of equilibria. Not only are the possible delays bounded, but, also, given any delay in government concession within these bounds, the equilibrium is unique.

To prove this, I first show existence for the lower and upper bounds, $\tau_0 = \underline{\tau}$ and $\tau_0 = \overline{\tau}$. The lower bound is straightforward, and the upper bound follows from a fixed-point argument that I explain below.

I define a modified problem in which the peak time τ_0 is chosen by a fictitious player and is given to both the citizens and the government. Suppose that time τ_0 is given. Recall that in the decay stage, the trajectory of participation is fixed at $\tilde{\pi}_t$. Thus, in equilibrium, citizens' exit times are given: a citizen with opportunity cost θ exits at time t, at which $F(\theta) = \tilde{\pi}_t$. Then, citizens' best reply is a sequence of entry times, given the government distribution of concessions and given their exit times.

Given these entry times, for any $t \geq \tau_0$, the government, in turn, must choose a hazard rate that makes the marginal agent indifferent between conceding and waiting another instant (in order to keep participation at the indifference level in the concession stage). The final step is to introduce the fictitious player whose only role is to adjust τ_0 for equation (9) to be satisfied with equality, given the government's best reply. This also allows me to get rid of discontinuities in the government's strategy at τ_0, and then I can apply standard fixed-point theorems. It is then straightforward to use the same fixed-point argument to show that for any $\tau_0 \in [\underline{\tau}, \overline{\tau}]$, an equilibrium exists.

Figures 1 and 2 illustrate the continuum of equilibria. In both figures, panel (a) shows the equilibrium with the shortest delay, $\underline{\tau}$; panel (b) shows an equilibrium with an intermediate delay, $\tau_0 \in (\underline{\tau}, \overline{\tau})$; and panel (c) shows an equilibrium with the maximum delay possible, $\overline{\tau}$.

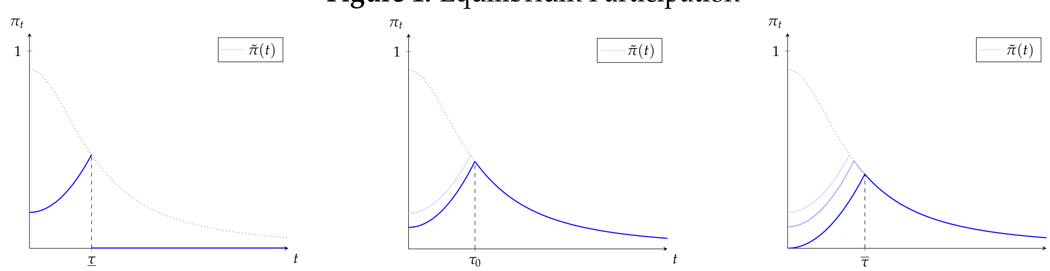
The three panels in Figure 1 illustrate the trajectory of participation for the three delays. The downward-sloping dotted line, $\tilde{\pi}_t$, corresponds to the indifference participation level. For any participation level π_t below this dotted line, the cost of the protest is still too low relative to the cost of the public good, and then the government is better off by ignoring protesters. Analogously, any participation level above this line is too costly, and then the government would rather concede. The three panels in Figure 2 show the distributions of government concession corresponding to each delay τ_0.

Note, first, that for any delay $\tau_0 \in [\underline{\tau}, \overline{\tau}]$, participation is increasing on $[0, \tau_0]$. This corresponds to the build-up stage. Since participation in this stage is everywhere below the line $\tilde{\pi}_t$, the government is better off by waiting. Then, in the three panels in Figure 2, $G(t) = 0$ on $[0, \tau_0)$. Once participation hits the dotted line, then the protest becomes too costly and the government has to make some concession. The very precise moment at which this happens corresponds to the *peak*. The equilibrium with the shortest delay, $\underline{\tau}$ in panel (a), corresponds to the one in which the government concedes with probability 1 at the peak. Then, the distribution of government concessions jumps up to 1, and everyone drops out.

In panel (b), $\tau_0 \in (\underline{\tau}, \overline{\tau})$. The government still makes a discrete concession, but with probability less than 1. Immediately after this concession, the government continues randomizing over time, and people continuously drop out. Participation coincides with the dotted line in equilibrium.

Note that as delay increases (moving to panels (b) and (c)), participation decreases for every t in the build-up stage. In Section A.1, I show that this is, in fact, a general feature of the equilibrium set.

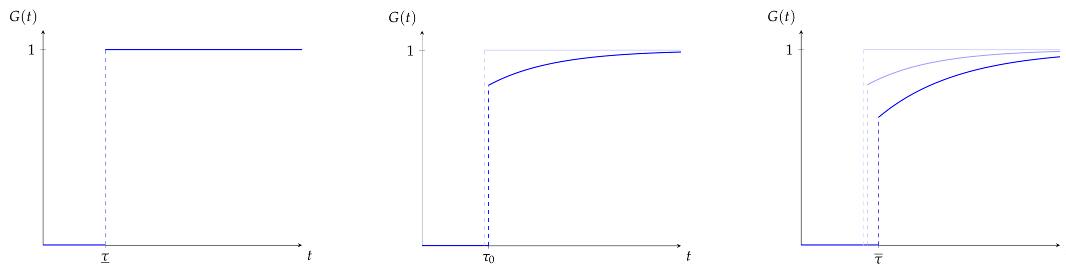
Figure 1. Equilibrium Participation



(a) Lower bound $\underline{\tau}$ (b) $\tau_0 \in (\underline{\tau}, \overline{\tau})$ (c) Upper bound $\overline{\tau}$

Before finishing this discussion, a brief comment on multiplicity is warranted. The existence of multiple equilibria is a natural feature of this model. As has been recognized in the literature, the spontaneous nature of mass uprisings gives them the features of a coordination problem that might, or might not, be successful (see Schelling (1960), Hardin (1997), and more recently, Bueno de Mesquita (2014)). In the case of static models of collective action, this implies that, in general, there are two equilibria in pure strategies: one in which a protest occurs and one in which it does not occur. In my model, not only

Figure 2. Government's Distribution of Concessions



(a) Lower bound $\underline{\tau}$ (b) $\tau_0 \in (\underline{\tau}, \overline{\tau})$ (c) Upper bound $\overline{\tau}$

we observe equilibrium with and without protests, but there is a continuum of equilibria in which a protest occurs. We can think of many reasons that a society's focal point centers on one particular equilibrium, such as social norms, culture, or coordination technologies. Despite their relevance, this model does not aim to explain these factors.

It is worth mentioning that the type of multiplicity observed here is insightful, in the sense that it provides key ideas about both the dynamics that are common to all equilibria and the trade-offs between persistence and participation across them. In the next section, I study how different equilibria within the equilibrium set compare to each other in terms of duration and participation.

5. Extensions

5.1. Government Partial Concessions. In many situations, the decision to provide a public good is not discrete. Authorities might make some concessions that do not entirely fulfill protesters' demands, but that dissuade some of them and, thus, alleviate the cost burden of the protest. In this section, I illustrate how the model can allow such concessions.

Suppose that the government can concede a fraction of the public good. Conceding a fraction α costs αq, where q is the cost of the entire public good. Every time the government concedes a fraction α of the public good, agents receive a flow utility $\alpha v(t - t_0)$ corresponding to their veteran payoff. Other than that, all payoffs remain the same as in the baseline case. The protest ends when either all citizens drop out, or the government has fully provided the public good.

The government's strategy is a function $h : [0, \infty) \to [0, 1]$ determining, for any time t, the additional share of the public good that the government provides at time t. Denote by $H(t)$ the share of the public good that has been provided at time t.

A citizen's payoff from entering at a time t_0 and exiting at t_1 is given by:

$$U(t_0, t_1; \theta) = -\theta \left[\frac{e^{-rt_0} - e^{-rt_1}}{r} \right] + \int_{t_0}^{t_1} e^{-rs} v(s - t_0) dH(s). \tag{10}$$

As in the main model, citizens' utility functions satisfy a single-crossing property, and their optimality conditions replicate the ones in the baseline model. Then, for any equilibrium $H : [0, \infty) \to [0, 1], (\pi_t^\sigma)_{t \geq 0}$ with $\tau_0 = \inf\{t \in [0, \infty] : h(t) > 0\}$, the following properties hold: (i) There is always delay in government concession—i.e., $\tau_0 > 0$; (ii) π_t^σ is continuous, increasing for $t \leq \tau_0$, and if $H(\tau_0) < 1$, decreasing for all $t \geq \tau_0$; (iii) The government makes, at most, one discrete concession at τ_0; and (iv) If $H(\tau_0) < 1$, then $H(t)$ is strictly increasing, concave, and for $t > \tau_0$ $H(t) < 1$.

5.2. Income and Opportunity Cost. So far, I have characterized agents' opportunity cost of the time spent in the protest by a parameter θ. This parameter captures the utility that agents give up by spending time on the protest instead of on other activities. In general, those other activities are often related to productive activities, and, thus, the opportunity cost can be associated with labor income.

To set ideas, consider a protest in which citizens have to decide whether and when to join. Every day, a citizen who joins the protest attends a demonstration that lasts one hour (every day is discrete, but consider this just an illustration). There is no physical cost of protesting, and the only cost to the citizen is the alternative use of that hour, which is equivalent to one hour-wage. Agents have heterogeneous income, ω, and let ϵ be the fraction of time an agent spends in the protest. In addition, there is a minimum level of consumption that citizens must satisfy, which corresponds to a subsistence level. We can think of this consumption as basic needs that the agent must fulfill before deciding whether to join a protest. I represent the subsistence level by a minimum income $\underline{\omega}$, such that any agent with income $\omega < \underline{\omega}$ cannot afford to become an activist.

The cost of attending the protest for a citizen with $\omega \geq \underline{\omega}$ is equivalent to:

$$\theta = u(\omega) - u\left(\omega\left(1 - \epsilon\right)\right), \tag{11}$$

where $u(\cdot)$ is the agent's utility of income (consumption). Then, the relation between citizens' income and opportunity costs depends on the shape of the utility function.

Consider, for instance, the following CRRA utility function:

$$u(\omega) = \frac{\omega^{1-\sigma}}{1 - \sigma} \tag{12}$$

for some $\sigma \geq 0$, $\sigma \neq 1$. In this case, the relation between income and opportunity cost depends on the curvature of the utility function, captured by σ. If $\sigma < 1$, then the marginal utility of income is increasing, which implies that for citizens with higher income, the hour spent demonstrating is more costly than for citizens with lower income. In the extreme case with $\sigma = 0$, utility is linear, and, thus, $\theta = \epsilon\omega$. In this case, there is a one-to-one relation between the distribution of opportunity costs and the distribution of income. In general, when the marginal utility of income is increasing, high-income citizens have greater incentives than those with lower incomes to delay their entry. If $\sigma > 1$, then the marginal utility of income is decreasing and high-income citizens are able to enter earlier.

There are other factors that might affect the relation between income and opportunity cost. For instance, job flexibility might affect how workers can make use of their own time. This, in general, is also related to education and the type of industries under analysis. Moreover, income might affect other factors in an agent's propensity to protest that might not be related to opportunity costs. Education, for instance, is key in how knowledgeable citizens are about the political environment. This implies that when comparing citizens with different income levels, we need to also take into account the effect of their income on their education levels.

5.3. Support for the Public Good. In this section, I illustrate the case in which only a subset of agents is willing to consider participating in the protest. Suppose that the value of the public good x is now a random variable that can take two values, 0 or 1, and let p be the probability of $x = 1$. Each citizen's value for the public good is independent of her opportunity cost.

Moreover, assume that citizens value the veteran prize only if they value the public good. Thus, I modify the veteran prize function to be $x \cdot v(t - t_0)$. With this new framework, agents who don't value the public good do not have incentives to participate in the protest. It is clear to see that the baseline case is equivalent to setting $p = 1$, and then as p decreases, the mass of citizens willing to enter the protest also decreases.

What is interesting about this perturbation is that all the dynamics of the model remain the same, but the set of equilibria is reduced. Citizens' problem is the same as in the baseline model, and their strategies can be characterized by thresholds $\tilde{\theta}_0(t), \tilde{\theta}_1(t)$. For any possible entry threshold $\tilde{\theta}_0(t)$, participation is just a rescaling of the original problem and is given by $\pi_t = p \cdot F(\tilde{\theta}_0(t))$. Thus, both Theorem 1 and Theorem 2 hold.

I highlight some of the main features that differentiate this case from the baseline case. Let $[\underline{\tau}, \overline{\tau}]$ be the equilibrium set in the baseline case, and denote by $[\underline{\tau}_p, \overline{\tau}_p]$ the equilibrium set with $p < 1$. First, note that it must be that $\underline{\tau} < \underline{\tau}_p$ and $\overline{\tau}_p \leq \overline{\tau}$. The intuition is analogous to the comparative statics in Proposition 1. Recall the lower bound corresponds to the equilibrium in which the government concedes with probability 1. When not all agents are willing to participate, it takes more time to make the government concede. The upper bound does not necessarily decrease, as it depends on the citizen with the lowest opportunity cost.

Consider, now, an intermediate equilibrium with delay $\tau_0 \in (\underline{\tau}_p, \overline{\tau}_p)$. Let $G(t)$ be the government distribution of concessions with $p = 1$, and $G_p(t)$ for $p < 1$. Note that both $G_p(t)$ and $G(t)$ have support $[\tau_0, \infty)$. Moreover, in both equilibria, participation must coincide on $[\tau_0, \infty)$. Then, the initial government concession is such that $G(\tau_0) < G_p(\tau_0)$.

The main idea of these differences is that the universe of citizens is smaller. However, conditional on reaching some participation level, those who are protesting have (weakly) higher opportunity costs than in the baseline case, and that makes them stronger in front of the government.

5.4. Refinements and Equilibrium Selection. For some problems, it might be relevant to refine the set of equilibria. The three main approaches that might be applied as refinements are: (i) reputation; (ii) global games; and (iii) coalition-proofness.

Reputational concerns in this model arise when there is some information about agents that is private. The attritional nature of the game makes behavioral types a la Abreu and Gul (2000) natural candidates for refinement. Introducing a probability of the government being a behavioral type that never concedes, and a probability of citizens being a type that will protest forever will pin down a unique equilibrium.[17]

Global games are a theoretical framework commonly used to study uprisings and regime change models. Since the seminal work of Morris and Shin (1998), their framework has been used to study public protests and revolutions in different institutional settings (see Edmond (2013), Egorov, Guriev and Sonin (2009), Boix and Svolik (2013) and Morris and Shadmehr (2018)). The key component in these models is a coordination game with incomplete information, in which uncertainty is generally about the strength of the regime. Although in static setups this pins down a unique equilibrium, this is not the case with dynamics. Angeletos, Hellwig and Pavan (2007) study the role of learning in a repeated framework, and they show that the dynamic nature of the game introduces multiplicity even under conditions that guarantee uniqueness in static games.

The possibility of coalition formation by citizens provides another rationale for equilibrium refinement. Naturally, political activism requires some organization that can be done before a protest begins. These pre-arrangements would make it possible to coordinate in an equilibrium with short delay by ensuring that a share of the population high enough would join the protest at the beginning. If all citizens were better off with this outcome,

[17]The issue with this refinement is that, depending on the parametrization, in this framework, it might not be very informative of the equilibrium selected.

we would expect this to be coalition-proof in the sense of Bernheim, Peleg and Whinston (1987) (see, also, Moreno and Wooders (1996) and Ray and Vohra (2001)).

Lastly, although in coalition-proofness agreements among agents are non-binding, sometimes leaders take some irreversible actions in order to obtain a specific outcome. In this framework, it could be the case that a leader wants to design a scheme to incentivize participation of people with higher opportunity costs, to make the government concede faster. [18]

6. Concluding Remarks

This paper presents a theory of the dynamics of participation in public protests. I develop a model in which a continuum of citizens protests to ask the government for a policy change or the provision of a public good. Citizens' participation is motivated by a psychological prize that they get when they win against the government. I show that any equilibrium in this dynamic game displays: (a) a *build-up* stage, during which citizens continuously join the protest and the government waits; followed by (b) a *peak*, at which participation reaches its maximum, and the government makes the first probabilistic concession; and, possibly, (c) a *decay* stage, in which people continuously drop out as the government concedes with some density. Also, when parametrized by the time at which the peak occurs, the set of possible values is bounded, and for each of them, the equilibrium is unique.

There are many directions in which these ideas might be developed. Probably the first and most important extension is to allow for heterogeneity in the value for the public good. Even when, in the model, this is just a renormalization, the questions that can be addressed by differentiating the cost from the value are empirically relevant. In particular, it would allow an understanding of how the relationship between people's participation costs and the value for the public good affect the duration of protests. Then, under the proper parametrization, we could understand whether issues that are more relevant to high-income people tend to be solved earlier than those of general interest.

People's heterogeneity in the value obtained from policies opens the door to other extensions, as well. If the government did not know the value of the policy and should learn it from the protest, this would give rise to novel equilibrium dynamics. In that case dynamic concessions are a double-edged sword, as conceding decreases the cost of the protest by persuading people to go home, but also decreases the information that the government can extract from it.

[18] Morris and Shadmehr (2018) construct a model in which a leader designs reward schemes that assign psychological rewards to citizens' actions. See also Bhavnani and Jha (2014) for the role of the leaders in organizing a movement in a nonviolent form.

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A. Appendix: Equilibrium Set and Comparative Statics

A.1. Trade-off between Persistence and Participation. The set of equilibria shows a trade-off between persistence and participation that is very insightful. First, there is an inverse relation between the peak in participation and delay in the first probabilistic concession: longer delay is consistent with a lower participation at the peak. This suggests that in a dynamic setting, the characterization of a successful protest should combine a *critical mass*, with a *critical persistence*. Ignoring this trade-off might result into an over-estimation of the critical mass required for a protest to be successful.[19] Second, there is also an inverse relation between initial participation and the peak time. Participation at time 0 reaches its maximum when delay is at the lower bound $\tau_0 = \underline{\tau}$, and its minimum when delay is at the upper bound $\tau_0 = \overline{\tau}$. Let $\underline{\pi}_0$ and $\overline{\pi}_0$ be the minimum and maximum levels of initial participation.

Corollary 1. *Fix an initial participation $\pi_0 \in [\underline{\pi}_0, \overline{\pi}_0]$. There exits a unique trajectory of participation $(\pi_t^\sigma)_{t\geq 0}$ with initial level $\pi_0^\sigma = \pi_0$.*

In other words, conditional on the first event, the trajectory of participation is unique. This gives an idea of how informative the first event of a social movement is with respect to the future trajectory of participation. Fixing the fundamentals, the initial participation is enough to describe the full trajectory of participation.

A.2. Expected Duration across Equilibria. How do different equilibria relate to each other in terms of welfare? In order to assess this, I first characterize equilibrium expected duration. From Theorem 2, it follows that the expected duration of a protest is increasing in τ_0. Putting this together with Corollary 1, I obtain the following.[20]

Corollary 2. *Expected duration increases with τ_0 and decreases with π_0.*

Even when duration varies monotonically along the equilibrium set, welfare analysis is more subtle. The government is always better off with the equilibrium with longest delay. For citizens, the result depends on the veteran reward. Consider, first, a situation in which we ignore the existence of the veteran prize. As citizens care about the public good, and protesting is costly for them, on aggregate, they will be better off with the equilibria with the shortest duration and the highest initial participation. Moreover, from agents' optimality condition, we learn that when the protest starts, in any equilibria but the upper

[19]This is in line with the intuition developed recently by Chenoweth and Belgioioso (2019), who propose that a protest can be described by its *momentum*, which is defined as a function of mass (i.e., participation), and velocity (i.e., the frequency of events).

[20]The result follows from the fact that the distributions of government concessions do not cross. Thus, the probability of a protest's survival is monotone in τ_0 for any $t > \tau_0$.

bound $\overline{\tau}$, there is a positive mass of agents who are strictly better off by entering the protest. That mass increases as the peak is reached sooner.

The veteran prize has a nontrivial effect. Activism is valuable to citizens even when protesting is costly. It might be that citizens are better off in an equilibrium with later concession because, then, they get to maximize their contribution to the social movement.

A.3. Changes in the Distribution of Opportunity Costs. An increase in citizens' opportunity costs has two effects. On the one hand, it directly affects agents' entry decision, as a citizen with a higher opportunity cost will want to wait longer. On the other hand, it has an indirect effect on the government's best response. As the opportunity cost of a citizen increases, the hazard rate that makes her drop out also increases. She becomes stronger in front of the government and force it to concede faster.

Consider, first, a general increase in agents' opportunity cost. When citizens' opportunity costs increase, it takes longer to reach the level of participation required to make the government concede with probability 1. This moves the lower bound of the equilibrium set to the right. If, instead, we apply a mean preserving spread to the distribution of opportunity costs, then the effect is ambiguous. The result now depends on the agent who is at the margin when the government is going to concede for sure. I formalize these ideas in the following result.

Proposition 1. *Let F_1 and F_2 be two symmetric and unimodal distributions, with corresponding equilibrium sets $[\underline{\tau}_1, \overline{\tau}_1]$ and $[\underline{\tau}_2, \overline{\tau}_2]$.*

(i) *If F_1 first-order stochastically dominates F_2, then $\underline{\tau}_1 \geq \underline{\tau}_2$.*
(ii) *If F_2 is a mean preserving spread of F_1, then $\underline{\tau}_1 > \underline{\tau}_2$ if and only if $v'(0) < \int \theta dF_1(\theta)$.*

This follows from the fact that the lower bound of the equilibrium set for a distribution F depends uniquely on $F(v'(0))$.[21] Then, changes to the costs distribution that increase the number of citizens willing to enter force the government to concede faster.

The effect of a change in opportunity costs over the upper bound is more subtle, as now the effect through the government's hazard rate plays a role. Consider a general increase in citizens' opportunity costs, so that protesting becomes more costly for every agent. Let F_1 be the initial distribution of opportunity costs and F_2 be the distribution after the increase. Then, $F_1(\theta) > F_2(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, and the following result holds.

Proposition 2. *Suppose that citizens' opportunity costs increase by the same proportion α, and let $[\underline{\tau}_\alpha, \overline{\tau}_\alpha]$ to be the new equilibrium set. Then, it must be that $\underline{\tau} < \underline{\tau}_\alpha < \overline{\tau}_\alpha < \overline{\tau}$.*

[21] In any equilibrium at which the government concedes for sure, the number of people who are willing to enter are those with opportunity cost $\theta \leq v'(0)$.

B. Appendix: Proofs

B.1. Proof of Theorem 1: Equilibrium Characterization. This section is devoted to prove the equilibrium characterization described in Section 2. I begin by proving some properties of the government's equilibrium strategy, which then I use to fully characterize the set of equilibria. In the first lemma, I show that if there is an interval after τ_0 in which the government does not concede (i.e. the distribution G is constant in that interval), then no agent who is in the protest drops out during that interval. More precisely, we say that an agent with opportunity cost θ is participating at a time t if $\sigma_t^\theta = 1$.

Lemma 1. *Assume $\tau_0 < \tau_1$ and take t_1, t_2 such that $\tau_0 \leq t_1 < t_2 \leq \tau_1$. If G is constant in (t_1, t_2), then no agent participating at t_1 drops out in $(t_1, t_2]$.*

Proof. For any citizen that is participating at t_1, she is strictly better off quitting at t_1, than at any $t \in (t_1, t_2]$. □

Lemma 2. *The support of G is either a singleton, or a connected interval $\mathcal{T} = [\tau_0, \tau_1]$.*

Proof. By contradiction, suppose there exists $t \in [\tau_0, \tau_1]$ such that $t \notin \mathcal{T}$. Then, $t > \tau_0$, and there exists $\epsilon \in (0, t - \tau_0]$ such that $G(t) - G(t - \epsilon) = 0$. But then $[t - \epsilon/2, t] \cap \mathcal{T} = \emptyset$, so if there is $t \notin \mathcal{T}$, there is an interval which does not belong to $\mathcal{T}$. Then take t_0, t_1, with $\tau_0 \leq t_0 < t_1 \leq \tau_1$ such that $G(s) = G(t_0)$ $\forall s \in [t_0, t_1)$.

Assume $[t_0, t_1)$ is maximal, i.e. there is no interval $[t_0', t_1')$ such that $[t_0, t_1) \subsetneq [t_0', t_1')$ and $G(s) = G(t_0')$ for every $s \in [t_0', t_1')$. Maximality of the interval implies that $t_0 \in \mathcal{T}$. If not, there exists $\epsilon_1 > 0$ such that $G(t_0) - G(t_0 - \epsilon_1) = 0$, but then $G(s) = G(t_0 - \epsilon)$ $\forall s \in [t_0 - \frac{\epsilon}{2}, t_1]$. By maximality, for every $\epsilon > 0$ $[t_1, t_1 + \epsilon) \cap \mathcal{T} \neq \emptyset$. Then, it is optimal for the government to concede at t_0 and at t_1.

Note that for the government to concede at t_0 the cost of conceding must less than or equal than the cost of waiting. The cost of conceding at t_0 is $\frac{q}{r}$, while the cost of waiting to concede at some $t_0 + \delta$ for $\delta > 0$ is given by

$$\int_0^\delta e^{-rs} c(\pi^\sigma_{t_0+s}, t_0 + s) ds + e^{-r\delta} \frac{q}{r} \tag{13}$$

Then, we have:

$$\int_0^\delta e^{-rs} c(\pi^\sigma_{t_0+s}, t_0 + s) ds + e^{-r\delta} \frac{q}{r} \quad \geq \quad \frac{q}{r} \qquad \forall \delta > 0 \tag{14}$$

or, equivalently

$$\int_0^{\delta} e^{-rs}\left(c(\pi^{\sigma}_{t_0+s}, t_0+s) - q\right) ds \quad \geq \quad 0 \qquad\qquad \forall \delta > 0. \tag{15}$$

Define $\bar{t} = \frac{t_0+t_1}{2}$. Note that as 15 holds for every $\delta > 0$, it must also hold for $\overline{\delta} = \bar{t} - t_0$.

Since $t_0 \in \mathcal{T}$, then it must be that $\pi^{\sigma}_{t_0} > 0$, as otherwise the cost of the protest is zero. Moreover, by lemma 1, no citizen drops out at $(t_0, t_1]$, so $\pi^{\sigma}_{\bar{t}+s} \geq \pi^{\sigma}_{t_0+s}$ for all $s \in (0, \overline{\delta}]$. As, $\pi_{t_0} > 0$, then the cost is strictly increasing in time, and we have:

$$c(\pi^{\sigma}_{t_0+s}, t_0+s) < c(\pi^{\sigma}_{\bar{t}+s}, \bar{t}+s) \qquad\qquad \forall s \in (0, \overline{\delta}] \tag{16}$$

Then, we can compute:

$$\begin{aligned}\int_{t_0}^{t_1} e^{-r(s-t_0)}\left(c(\pi^{\sigma}_s, s) - q\right) ds \quad &= \quad \int_{t_0}^{\bar{t}} e^{-r(s-t_0)}\left(c(\pi^{\sigma}_s, s) - q\right) ds \\ &+ \quad e^{-r(\bar{t}-t_0)} \int_{\bar{t}}^{t_1} e^{-r(s-\bar{t})}\left(c(\pi^{\sigma}_s, s) - q\right) ds\end{aligned} \tag{17}$$

The first term on the right hand side is weakly greater than 0. By 16 the second term must then be strictly greater than zero, which implies $\int_{t_0}^{t_1} e^{-rs}\left(c(\pi^{\sigma}_{t+s}, t+s) - q\right) ds > 0$. But then the government strictly prefers to concede at t_0 than at t_1, which is a contradiction. □

Lemma 3. *If $\mathcal{T}$ is not a singleton, then it must be that $c(\pi^{\sigma}_s, s) = q$ and $\pi^{\sigma}_s = \tilde{\pi}_s$ for every $s \in [\tau_0, \tau_1)$.*

Proof. For the government to be randomizing over concession times $\tau \in [\tau_0, \tau_1]$, it must be that:

$$\int_0^{\tau} e^{-rs} c(\pi^{\sigma}_s, s) + e^{-r\tau}\frac{q}{r} = a \qquad\qquad \forall \tau \in [\tau_0, \tau_1] \tag{18}$$

for some constant a. Taking first order conditions with respect to τ, we obtain $c(\pi^{\sigma}_{\tau}, \tau) - q = 0$, which proves the result. □

Lemmas 2 and 3 provide a characterization of the regions over which the government concedes. The problem for the government is a stopping time, in which I allow it to randomize. For citizens the problem is a little different. Given that I do not impose restrictions on the actions that citizens can take, they could enter and exit the protest many times. So far there is nothing that prevents a citizen to protest over a time interval, then drop out to spend some time outside the protest, and then protesting again. However, I show that in equilibrium citizens enter and exit at most once. In particular, their optimality conditions satisfy a monotonicity property with respect to opportunity cost, that ensures that citizens' strategies can be characterized by opportunity cost thresholds. In Lemma 4 I give sufficient conditions for these entry and exit times to be optimal. Optimality conditions are stated in terms of the hazard rate of government concession, $\lambda_t = \frac{g(t)}{1-G(t)}$, which corresponds to the instantaneous probability of government concession conditional on the it being still in the game.

Lemma 4. *In equilibrium, citizens enter and exit at most once. For a person with opportunity cost θ who does enter, the optimal entry and exit times, $t_0(\theta), t_1(\theta)$ are a solution to the following sufficient conditions:*

$$\theta = \lambda_{t_1} v(t_1 - t_0) \tag{19}$$

$$\theta = \frac{1}{1-G(t_0)} \int_{t_0}^{t_1} e^{-r(s-t_0)} v'(s-t_0) dG(s) \tag{20}$$

Moreover, optimal entry and exit times satisfy $t_0'(\theta) > 0$ and $t_1'(\theta) < 0$, respectively.

Proof. Consider a citizen with opportunity cost θ who is planning to enter, on the equilibrium path, at some time t_0 and exit at t_1, i.e. $\sigma_t^\theta = 1$ for $t \in [t_0, t_1)$. Given a random concession time τ for the government, the citizen solves the following problem:

$$\max_{(t_0,t_1)\in[0,\infty]^2} E\left[-\theta \int_{t_0}^{t_1 \wedge \tau} e^{-rs} ds + e^{-r\tau} \mathbb{1}_{\tau < t_1} v(t - t_0)\right] \tag{21}$$

where the expectation is taken over τ, and where we have omitted additive payoffs that are not under the agent's control. Plugging in the distribution of government concessions G the objective function can be rewritten as:

$$\begin{aligned} U(t_0, t_1; \theta) &= \int_{t_0}^{t_1} \left[-\frac{\theta}{r}(e^{-rt_0} - e^{-rs}) + e^{-rs} v(s - t_0)\right] dG(s) \\ &\quad -(1 - G(t_1))\frac{\theta}{r}(e^{-rt_0} - e^{-rt_1}) \end{aligned} \tag{22}$$

As long as an agent is in the protest she has to pay the cost of the protest. If the government concedes before the time she drops out, the citizen gets the veteran reward. If the government has not conceded by the time the agent drops (which happens with probability $(1 - G(t_1)))$, then the agent only pays the cost of the protest and does not get any prize. Taking first order conditions with respect to t_0 and t_1, we have:

$$\frac{\partial U}{\partial t_0} = -(1 - G(t_0))\theta + g(t_0)v(0) + \int_{t_0}^{t_1} e^{-r(s-t_0)} v'(s - t_0) dG(s) \tag{23}$$

$$\frac{\partial U}{\partial t_1} = -\theta e^{-rt_1}(1 - G(t_1)) + g(t_1) e^{-rt_1} v(t_1 - t_0) \tag{24}$$

Reorganizing, we obtain equations 19 and 20 from the lemma. Note that these equations have a unique solution.

The fact that first order conditions are also sufficient follows from a single-crossing property of agents utility with respect to opportunity cost. In particular, the marginal utilities of agents' strategies are monotone in θ, i.e.

$$\frac{\partial^2 U}{\partial t_0 \partial \theta} = e^{-rt_0}(1 - G(t_0)) \geq 0 \qquad \frac{\partial^2 U}{\partial t_1 \partial \theta} = -e^{-rt_1}(1 - G(t_1)) \leq 0 \tag{25}$$

Thus, citizens follow monotone strategies satisfying $t_0'(\theta) > 0$, $t_1'(\theta) < 0$.

Now, suppose an agent is considering to reenter. Note that once the agent exits, her problem becomes the same from equation 21, as the veteran payoff goes back to zero. But then by the single crossing property we just proved reentry cannot be optimal. This concludes the proof. □

From equation 19 we see that an agent will exit when the marginal cost of staying another instant, i.e. θ, exceeds the marginal benefit, i.e. the prize times the instantaneous probability of government concession conditional on the government being still in the game. Equation 20 has a similar interpretation: the agent enters if the marginal cost is smaller than the marginal benefit. The marginal benefit now has two components. The first term in the right hand side captures the probability of obtaining the prize immediately, while the second one corresponds to the marginal benefit obtained from increasing the prize for all future periods that the agent plans to protest.

From Lemma 4, at any time agents' decision can be characterized by opportunity cost thresholds. More precisely, define $\tilde{\theta}_0(t) = t_0^{-1}(t)$, and note that this corresponds to the agent who is indifferent between entering at time t or waiting (i.e. equation 20 holds with equality). Any citizen with opportunity cost $\theta < \tilde{\theta}_0(t)$ is strictly better off by being in the protest. Analogously define $\tilde{\theta}_1(t) = t_1^{-1}(t)$, and note that it corresponds to the agent who is indifferent between staying in the protest another instant or exit immediately. Any citizen with $\theta > \tilde{\theta}_1(t)$ is strictly better off by dropping out.

We now put this ingredients together to prove Proposition 1 using the following steps.

Step 1: If $\tau_0 < \tau_1$, then π_t^σ is strictly decreasing in t, for every $t \in [\tau_0, \tau_1)$. From lemma 3, it must be that $c(\pi_t^\sigma, t) = q$ at every $t \in [\tau_0, \tau_1)$. Then, $\pi_t = \tilde{\pi}(t)$ for every $t \in [\tau_0, \tau_1)$. This function is well-defined, continuous and decreasing by assumption 1.

Step 2: The distribution has at most one discrete jump at τ_0. Suppose there is $t > \tau_0$ such that the distribution G jumps at t, i.e. there is $\epsilon > 0$ such that $G(t) > G(s)$ for all $s \in [t - \epsilon, t)$.

But then there is an interval over which citizens will not drop, contradicting the previous step.

Step 3: If $\tau_0 < \tau_1$, then at every $t \in [\tau_0, \tau_1)$ the distribution of concessions G has decreasing hazard rate. From equation 19 in Lemma 4, for citizens' decision to be optimal the exit threshold must satisfy:

$$\tilde{\theta}_1(t) = \lambda_t v(t - t_0(\tilde{\theta}_1(t))) \tag{26}$$

From the previous step, we have that the threshold must satisfy $F(\tilde{\theta}_1(t)) = \tilde{\pi}(t)$, and then it is decreasing over time. Then the left-hand side of equation 26 is decreasing, while the prize function increases over time, so it has to be that λ_t is decreasing.

Step 4: If $\tau_0 < \tau_1$, then $\tau_1 = \infty$. Suppose $\tau_1 < \infty$. First, it must be that $G(\tau_1) = 1$. Suppose that this is is not the case and the government stops conceding at some τ with $G(\tau) < 1$. Using the same arguments as in the proof of lemma 2, it must be $c(\pi_\tau, \tau) \geq q$. But then $\pi_\tau > 0$, as otherwise $c(\pi_\tau, \tau) = 0$ by assumption 1. By lemma 1 no citizen drops after τ, but then as the cost is increasing in time, eventually the cost of the protest would be higher than the cost of waiting, contradicting the optimality of the government's strategy. Thus, it must be that $G(\tau_1) = 1$. If this is the case, it must be that $\int_0^{\tau_1} \lambda_s ds = \infty$, which cannot happen in finite time as λ_t is decreasing in t. So, $\tau_1 = \infty$.

Step 5: If a citizen with opportunity cost θ ever enters the protest (i.e. $\exists t$ such that $\sigma_t^\theta = 1$), then $t_0(\theta) \leq \tau_0 \leq t_1(\theta)$. $t_1(\theta) \geq \tau_0$ follows directly from optimality, as otherwise the expected prize is zero with probability 1. Now consider an agent with opportunity cost θ entering at $t_0 > \tau_0$. From lemma 4, the marginal benefit of entering is given by:

$$\lambda_{t_0} v(0) + \frac{1}{1 - G(t_0)} \int_{t_0}^{t_1} e^{-r(s-t_0)} v'(s - t_0) dG(s) \tag{27}$$

By step 3, the expression above is decreasing in t_0 for any $t_0 \geq \tau_0$, and the marginal cost is constant. Then, the agent is strictly better off entering earlier.

Step 6: At any $t < \tau_0$, π_t^σ is increasing. From the previous claim, $\pi_t^\sigma = F(\tilde{\theta}_0(t))$, which is increasing.

Step 7: π_t^σ is continuous at every $t \in [0, \infty]$. We know that π_t^σ is continuous on $[\tau_0, \infty]$, and by the entry condition we also know it is continuous in $[0, \tau_0)$. It remains to show that it is also continuous at τ_0. In particular, we rule out cases in which there is a positive mass of people entering at a given time t (see figure 3). Take two agents entering at a given time $\bar{t}_0$. Note that as $\tilde{\pi}_t$ is strictly decreasing, these two agents cannot exit at the same time. Suppose they exit at some times $t_1 < t_1'$. Thus, from the exit condition their opportunity costs are given by $\tilde{\theta}_1(t_1) > \tilde{\theta}_1(t_1')$. But from the entry condition, we have:

$$\tilde{\theta}_1(t_1) = \int_{\bar{t}_0}^{t_1} e^{-r(s-\bar{t}_0)} v'(s-\bar{t}_0) dG(s) < \int_{\bar{t}_0}^{t_1'} e^{-r(s-\bar{t}_0)} v'(s-\bar{t}_0) dG(s) = \tilde{\theta}_1(t_1') \tag{28}$$

a contradiction.

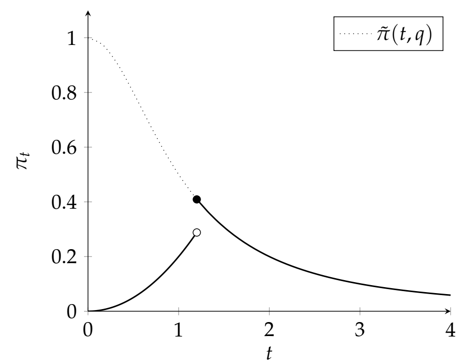

Figure 3. Continuity of π_t.

Step 8: $\tau_0 > 0$. We begin by showing that if $G(\tau_0) = 1$, then $\tau_0 > 0$. For $\tau_0 \in \mathcal{T}$ it must be that $c(\pi_{\tau_0}, \tau_0) \geq q$ (see the proof of lemma 2). The benefit of the last citizen entering is given by $G(\tau_0) \cdot v'(0)$, and then, for this to be an equilibrium, it must be that $F(v'(0)) = \tilde{\pi}_{\tau_0}$. By assumption 3, this time must be strictly positive.

Denote by $\underline{\tau}$ the time at which the government concedes with probability 1. Then, we prove that if $G(\tau_0) < 1$, then it must be that $\tau_0 > \underline{\tau}$. Note that if $G(\tau_0) < 1$ then by lemma 3 it must be that $c(\pi_{\tau_0}, \tau_0) = q$. The payoff to the last agent entering is given by $G(\tau_0)v'(0)$, and then it must be that at $F\left(G(\tau_0)v'(0)\right) = \tilde{\pi}_{\tau_0}$. But $\tilde{\pi}_{\tau_0} < \tilde{\pi}_{\underline{\tau}}$, so by assumption 1 it must be $\tau_0 > \underline{\tau}$.

Step 9: In equilibrium the government concedes in finite time, i.e. $\lim_{t\to\infty} G(t) = 1$. From step 4, $\tau_1 = \infty$. Denote by $\underline{\lambda}_t = \frac{\underline{\theta}}{v(t)}$ the hazard rate that makes the lowest opportunity cost citizen indifferent between dropping out and protesting at any time t. Note that by assumption 2, $\underline{\lambda}_t > 0$ for all t. Moreover, $\lambda_t \geq \underline{\lambda}_t$ for all t, and then $\int_0^\infty \lambda_t dt \to \infty$. So we have:

$$\lim_{t\to\infty} G(t) = 1 - \lim_{t\to\infty} \left[(1 - G(\tau_0)) \exp\left(- \int_0^t \lambda_s ds \right) \right] = 1 \tag{29}$$

With this, we complete the proof of Theorem 1. $\square$

Lemma 5. *Government initial concession $G(\tau_0)$ is decreasing in τ_0.*

B.2. Proof of Lemma 5. Using Lemma 5, the enttry threshold can be written as:

$$\tilde{\theta}_0(t) = \begin{cases} \int\limits_t^{t_1} e^{-r(s-t)} v'(s-t) dG(s) & t \in [0, \tau_0) \\ \tilde{\theta}_1(t) & t = \tau_0 \end{cases} \tag{30}$$

Using continuity of π_t, it has to be that $\tilde{\theta}_0(t)$ is also continuous, i.e. $\lim\limits_{t \to \tau_0^-} \tilde{\theta}_0(t) = \tilde{\theta}_1(\tau_0)$. Thus, at τ_0 the following condition holds:

$$\tilde{\theta}_0(\tau_0) = v'(0)G(\tau_0) \quad \Rightarrow \quad G(\tau_0) = \frac{\tilde{\theta}_1(\tau_0)}{v'(0)}$$

□

B.3. Proof of Theorem 2: A Continuum of Equilibria. It is direct to see that there is an equilibrium with $\tau_0 = \underline{\tau}$. I begin by showing that there exists an equilibrium satisfying $\tau_0 = \overline{\tau}$. Then, I prove that for any τ_0 in between this thresholds, an equilibrium exists.

Lemma 6. *There exists an equilibrium $(G, (\pi_t^\sigma)_{t \geq 0})$ with $\tau_0 = \overline{\tau}$ satisfying*

$$\underline{\theta} = \int_{\overline{\tau}}^{\infty} e^{-rs} v'(s) dG(s) \tag{31}$$

Proof. In order to prove existence of this equilibrium with the longest delay, I show that there exists a fixed point satisfying condition 31. As I describe in Section 4.2, in equilibrium citizens' exit times are determined by the government indifference condition.[22] Then, given their exit times and the government distribution of concessions $G(t)$, their best reply associates each exit time $t \in [\tau_0, \infty)$, with an entry time $t_0(t)$. The government, given these entry times chooses a distribution of concessions $G(t)$.

I consider a modified game, in which a fictitious player chooses the delay τ_0, in such a way that, given $G(t)$, condition 31 is satisfied. In this modified game, the government, given citizens' and the fictitious player's best responses, chooses a probability distribution of concessions for $G(t)$, for any $t \in [\tau_0, \infty)$. Citizens, given the distribution of the government, choose their entry times.

Define the best reply correspondence: $\Psi : \mathbf{Z} \twoheadrightarrow \mathbf{Z}$ with tipical element $\mathbf{z} = (G, t_0, \tau_0)$ as:

$$\Psi = (\Gamma(t_0, \tau_0), \Phi(G, \tau_0), \Theta(G, t_0)) \tag{32}$$

where $\Gamma(t_0, \tau_0)$ is the government's best reply, $\Phi(G, \tau_0)$ is citizens' best reply, and $\Theta(G, t_0))$ is the best reply of the fictitious player.

[22]More precisely, on the support $\mathcal{T}$, it has to be the case that $\pi_t = \tilde{\pi}_t$. Thus, there exist a unique exit threshold $\tilde{\theta}_1(t)$ such that $\tilde{\pi}_t = F(\tilde{\theta}_1(t))$ for every $t \in \mathcal{T}$.

The space $\mathbf{Z} = [0,T] \times \mathcal{S} \times \mathcal{C}$ is such that $\mathcal{S}$ corresponds to the space of probability distributions,[23] and $\mathcal{C}$ corresponds to the space of continuous functions. T is the upper bound on the maximum concession time $\overline{\tau}$. I use Kakutani-Fan-Glicksberg theorem to prove that an equilibrium exists. This theorem states that if $\mathbf{Z}$ is a nonempty compact convex subset of a locally convex Hausdorff space, and the correspondence $\Psi : \mathbf{Z} \twoheadrightarrow \mathbf{Z}$ has closed graph and nonempty convex values, then the set of fixed points is compact and nonempty (Aliprantis and Border (2013), Corollary 17.55).

Step 1: Define Citizens' Best Response $\Phi : [0,T] \times \mathcal{S} \to \mathcal{C}$. In equilibrium, given a distribution $G \in \mathcal{S}$ with support $[\tau_0, \infty)$, for each possible exit time $t \in [\tau, \infty]$, $t_0(t)$ is the optimal entry time that solves the following equation:

$$\tilde{\theta}_1(t) = \int_{t_0}^{t} e^{-r(s-t_0)} v'(s-t_0) dGs \tag{33}$$

where $\tilde{\theta}_1(t) = F^{-1}(\tilde{\pi}(t))$. Figure 4 illustrates citizens' best reply function.

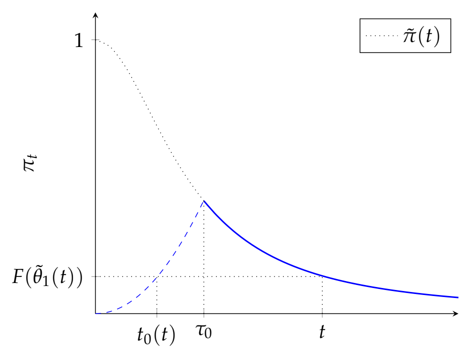

Figure 4. Citizens' exit is determined by $\tilde{\pi}_t = F(\tilde{\theta}_1(t))\ \forall t \in [\tau_0, \infty)$. A citizen with opportunity cost $\theta = \tilde{\theta}_1(t)$ exits at t, and given this exit time, equation 33 defines the entry time $t_0(t)$.

Step 2: Define Government's Best Response $\Gamma : \mathcal{C} \times [0,T] \to \mathcal{S}$. In equilibrium, given citizens' best reply $t_0 \in \mathcal{C}$ and the delay time τ_0, the government chooses a distribution of concessions over $[\tau_0, \infty)$, i.e. $G : [\tau_0, \infty) \to [0,1]$ such that:

$$G(t) = 1 - (1 - G(\tau_0)) \exp\left(-\int_{\tau_0}^{t} \lambda_s ds \right) \tag{34}$$

with $G(\tau_0) = \frac{\tilde{\theta}_1(\tau_0)}{v'(0)}$, and $\lambda_t = \frac{\tilde{\theta}_1(t)}{v(t - t_0(t))}$.

[23] Space of functions that are increasing, right-continuous, and such that $\lim_{t \to -\infty} G(t) = 0$ and $\lim_{t \to \infty} G(t) = 1$.

Step 3: Fictitious Player Best Response. The fictitious player best response $\Theta : \mathcal{C} \times \mathcal{S} \to [0, T]$ chooses a time $\tau_0 \in [0, T]$, that solves

$$\underline{\theta} = \int_{\tau_0}^{\infty} e^{-rs} v'(s) dG(s) \tag{35}$$

Step 4: **Z** *is a non-empty, convex and compact subset of a locally convex Haussdorf space.*

Let T be the time at which $\underline{\theta} = e^{-rT} v'(T)$. This is the time that makes the lowest opportunity cost citizen indifferent of entry when the government concedes for sure, which satisfies $T > \overline{\tau}$. Thus, $[0, T]$ is well defined, compact, and convex. Since both $G(\tau_0)$ and λ_t are continuous and well defined, the space of government's distribution of concession is non-empty. The function G constrained to $[\tau, \infty)$ is continuous and bounded. Moreover, they are monotone by Proposition 1, and have bounded variation. By Helly's selection theorem, it is also compact.

Similarly, t_0 is a monotone continuous function with values in $[0, \tau_0)$, and then it has bounded variation. Moreover, it is uniformly bounded, and then we can apply Helly's selection theorem to obtain compactness. To see that it is non-empty, fix t, and note that $t_0(t)$ solves the following equation:

$$\tilde{\theta}_1(t) = \int_{t_0}^{t} e^{-r(s-t_0)} v'(s - t_0) dGs \tag{36}$$

which has always a unique solution for every $t \in [\tau, \infty]$. By Tycohnoff Product Theorem (see Aliprantis and Border (2013), Theorem 2.61), the space **Z** is compact in the product topology.

Step 5: Ψ has closed graph. Take a sequence $(t_0^n, G^n, \tau^n) \in Graph(\Psi)$ such that $(t_0^n, G^n, \tau_0^n) \to (\bar{t}_0, \overline{G}, \overline{\tau}_0)$. We want to show $(\bar{t}_0, \overline{G}, \overline{\tau}_0) \in Graph(\Psi)$.

Claim 1. Γ has closed graph. We show that for any sequence $(\tau_0^n, G^n, t_0^n) \to (\overline{\tau}_0, \overline{G}, \bar{t}_0)$, with $(\tau_0^n, G^n) \in \Gamma(t_0^n)$ for all n, then $(\overline{\tau}_0, \overline{G}) \in \Gamma(\bar{t}_0)$.

Note that by continuity of $\tilde{\theta}_1(t)$, $G^n(\tau_0^n) \to \overline{G}(\overline{\tau}_0)$. Moreover, by continuity of v and F^{-1} the hazard rate $\lambda_n(t)$ converges uniformly to:

$$\overline{\lambda}(t) = \frac{F^{-1}(\tilde{\pi}(t))\epsilon}{v(t - \bar{t}_0(t))} \tag{37}$$

which proves the graph is closed.

Claim 2. Φ has closed graph. We show that for any sequence $(\tau_0^n, G^n, t_0^n) \to (\overline{\tau}_0, \overline{G}, \bar{t}_0)$, with $t_0^n \in \Phi(\tau_0^n, G^n)$ for all n, then $\bar{t}_0 \in \Phi(\overline{\tau}_0, \overline{G})$.

Rewrite t_0 as the solution to a fixed point problem to the following equation:

$$H(t_0; G, \tau_0) = \frac{1}{r}\left[\ln F^{-1}(\tilde{\pi}(t)) - \ln\left(\int_{t_0}^{t} e^{-rs} v'(s - t_0(t)) dG(s)\right)\right] \tag{38}$$

Thus, it is enough to prove that $\|\bar{t}_0 - H(\bar{t}_0)\| = 0$. Note that:

$$\|\bar{t}_0 - H(\bar{t}_0; \overline{G}, \overline{\tau})\| \leq \|\bar{t}_0 - t_0^n\| + \|t_0^n - H(t_0^n)\| + \|H(t_0^n; G^n, \tau^n) - H(\bar{t}_0; \overline{G}, \overline{\tau})\| \tag{39}$$

the first two terms in the right-hand side converge to 0 by hypothesis. The third one also converges pointwise to 0 as $\int_{t_0^n}^{t} e^{-rs} v'(t - t_0^n(t)) dG^n(s) \rightarrow \int_{\bar{t}_0}^{t} e^{-rs} v'(t - \bar{t}_0(t)) d\overline{G}(s)$ for all t.

Claim 3. Θ has closed graph. We show that for any sequence $(\tau_0^n, G^n, t_0^n) \to (\overline{\tau}_0, \overline{G}, \bar{t}_0)$, with $(\tau_0^n) \in \Theta(t_0^n, G^n)$ for all n, then $(\overline{\tau}_0) \in \Gamma(\bar{t}_0, \overline{G})$. Note that G^n converges to $\overline{G}$ in distribution, and then applying Continuous Mapping Theorem we obtain

$$\int_{\tau_0^n}^{\infty} e^{-rs} v'(s) dG^n(s) \to \int_{\overline{\tau}_0}^{\infty} e^{-rs} v'(s) d\overline{G}(s) \tag{40}$$

Then, using claims 1, 2 and 3, we have that Ψ has closed-graph, and therefore is upper-hemicontinuous. By Kakutani-Fan-Glicksberg theorem it has a fixed point. □

Lemma 7. *Let $(G^1, (\pi_t^1)_{t\geq 0})$ and $(G^2, (\pi_t^2)_{t\geq 0})$ be two distinct equilibria with delays τ_0^1, τ_0^2, such that $\tau_0^1 < \tau_0^2$. Then, the distributions of concessions G^1, G^2 do not cross at any $t \in [\tau_1, \infty]$.*

Proof. From agents entry condition, we have

$$\frac{\partial \tilde{\theta}_0(t)}{\partial \tau_0} = -e^{-r(\tau_0 - t)} v'(\tau_0 - t) g(\tau) + \int_t^{t_1(t)} e^{-r(s-t)} v'(\tau_0 - t) g'(\tau_0) ds < 0 \tag{41}$$

Given that this holds for all $t \in [0, \tau_0)$, the functions $t_0(t)$ do not cross, and this ensures the hazard rates do not cross, and then the distributions of concessions do not cross either. □

We can now show that any $\tau_0 \in [\underline{\tau}, \overline{\tau}]$ generates an equilibrium. Fix an arbitrary $\tau^* \in (\underline{\tau}, \overline{\tau})$ and let $(G, (\pi_t)_{t\geq 0})$ be the equilibrium consistent with it. From lemma 7,

$$\underline{\theta} < \int_{\overline{\tau}}^{\infty} e^{-rs} v'(s) dG(s) \tag{42}$$

and $G(\tau) < 1$. Then we can solve the same fixed point problem from the previous claim fixing the fictitious player strategy to choosing τ^*. Using the same arguments, a fixed point exists. As τ^* was arbitrary, this completes the proof. □

B.4. Proof of Proposition 1. Recall that for any distribution of opportunity costs F_j, the lower bound $\underline{\tau}_j$ is given by the equilibrium in which the government concedes with probability 1, and then it is such that $\tilde{\pi}_{\underline{\tau}_j} = F_j(v'(0))$. Then, (i) follows from the fact that F_1 first order stochastically dominates F_2, and then $F_1(v'(0)) < F_2(v'(0)$. To prove (ii) and (iii), note that as F_1 is symmetric and unimodal and F_2 is obtained from a mean preserving spread, then $F_2(\theta) < F_1(\theta)$ for every $\theta < \int \theta dF_1(\theta)$, and $F_2(\theta) > F_1(\theta)$ otherwise.

B.5. Proof of Proposition 2. The effect of the proportional increase in opportunity costs follows directly from part (i) in Proposition 1. Now we show $\overline{\tau}_\alpha < \overline{\tau}$. Denote the initial government strategy by G, which is given by:

$$G(t) = 1 - (1 - G(\tau_0)) \exp\left(-\int_0^t \lambda_s ds\right) \tag{43}$$

with $\lambda_s = \frac{\tilde{\theta}(t)}{v(t - t_0(\tilde{\theta}(t)))}$. Keeping government concession constant, an increase in agents' opportunity costs delays agents' optimal entry times. As all citizens' opportunity costs increase in the same proportion, then it must be that the new exit threshold is proportional to the initial one, $\tilde{\theta}_1'(t) = \tilde{\theta}_1(t)$. Consider first a compensated movement in the government's distribution of concessions such that citizens' entry times with the new opportunity costs, are equal to the original ones. The new hazard rate is given by $\lambda_t' = \frac{\tilde{\theta}'(t)}{v(t - t_0(\tilde{\theta}(t)))} = \lambda_t(1 + \alpha)$. Given that agents entry times remain the same, $\overline{\tau}$ remains the same, and the initial probability of government concession must increase to $G'(\overline{\tau}) = \frac{\tilde{\theta}(\overline{\tau})}{v'(0)} = \frac{\tilde{\theta}(\overline{\tau})(1+\alpha)}{v'(0)} = (1 + \alpha)G(\overline{\tau})$.

Fix an exit time t, so that a citizen with cost $\tilde{\theta}(t)$ exits at t and enters at time t_0. We're constructing an equilibrium in which this same citizen, now with opportunity cost $\tilde{\theta}'(t) = (1 + \alpha)\tilde{\theta}(t)$ enters and exits at the same times. From their entry conditions, we have the following:

$$\tilde{\theta}'(t) = \int_0^t e^{-r(s-t_0)} v'(s - t_0) \lambda_s' (1 - G'(\overline{\tau})) \exp\left(-\Lambda'(s)\right) ds \tag{44}$$

$$\tilde{\theta}(t)(1 + \alpha) = \int_0^t e^{-r(s-t_0)} v'(s - t_0) \lambda_s (1 + \alpha)(1 - G'(\overline{\tau})) \exp\left(-\Lambda(s)(1 + \alpha)\right) ds \tag{45}$$

$$\tilde{\theta}(t) = (1 - G'(\overline{\tau})) \int_0^t e^{-r(s-t_0)} v'(s - t_0) \lambda_s \exp\left(-\Lambda(s)(1 + \alpha)\right) ds \tag{46}$$

where $\Lambda(t) = \int_0^t \lambda_s ds$, and $\Lambda'(t) = \int_0^t \lambda_s' ds$. The left-hand side of equation 46 is equal to

$$\tilde{\theta}(t) = (1 - G(\overline{\tau})) \int_0^t e^{-r(s-t_0)} v'(s - t_0) \lambda_s \exp\left(-\Lambda(s)\right) ds \tag{47}$$

which is strictly greater than the right hand side of 46. Then keeping the same entry times is not an equilibrium. Citizens have incentives to delay their entry, which will increase the government hazard rate. As we picked an arbitrary agent, this is also the case for the citizen with the lowest opportunity cost. Then it must be that $\overline{\tau}' < \overline{\tau}$. $\square$